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General Certificate of Education (A-level)
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Mathematics MFP4

(Specification 6360)

Further Pure 4

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{vmatrix} \mathbf{i} & 2 & -2 \\ \mathbf{j} & 2 & -2 \\ \mathbf{k} & 3 & 5 \end{vmatrix} = \begin{pmatrix} 16 \\ -16 \\ 0 \end{pmatrix}$	B1		$\overrightarrow{AB}$ or $\overrightarrow{AC}$ correct
		M1		Attempt at cross product – at least one component correct
		A1	3	All correct components – no further working seen or attempted
(b)	$16:-16:\ 0 = 1:-1:\ 0$ Cartesian equation is $x - y =$ constant	M1		Correct structure using their perpendicular from a) : $ax+by+cz=d$
	Use $\mathbf{c} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} \Rightarrow$ constant $= -1$	A1	2	Finding correct value of d - CAO
	$\therefore\ x - y = -1$			
	Alternative:			
	$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} + \lambda\begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}$			
	$x = 1 + 2\mu - 2\lambda$ ① $y = 2 + 2\mu - 2\lambda$ ②	(M1)		Correct parametric structure with an attempt to eliminate variables
	$z = -1 + 3\mu + 5\lambda$ ③			
	① – ② $\Rightarrow x - y = -1$	(A1)	(2)	
(c)	$\overrightarrow{AD} = \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$	B1		Obtaining $\overrightarrow{AD}$ or $\overrightarrow{DA}$
	$\overrightarrow{AB}\times\overrightarrow{AC}\,.\,\overrightarrow{AD} = \begin{pmatrix} 16 \\ -16 \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix}$	M1		Scalar product with their $\overrightarrow{AD}$ and answer from part (a) or determinant seen $\overrightarrow{AB}\times\overrightarrow{AC}\,.\,\overrightarrow{AD} = \begin{vmatrix} 2 & -2 & 3 \\ 2 & -2 & -1 \\ 3 & 5 & -1 \end{vmatrix}$
	$= 64$ (cubic units)	A1	3	CAO
	Total		**8**	

Q	Solution	Marks	Total	Comments
2(a)	$\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 3 \\ 1 & 2 & -3 & 4 \\ 2 & 1 & a & b \end{array}\right]$			
	$r_3 \to r_3 - r_1$ $r_2 \to 2r_2 - r_1$ $\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 3 \\ 0 & 5 & -5 & 5 \\ 0 & 2 & a+1 & b-3 \end{array}\right]$	M1		Correct row operations used to create two zeros in first column – coefficients must be correct
	$r_3 \to 5r_3 - 2r_2$ $\left[\begin{array}{ccc\|c} 2 & -1 & -1 & 3 \\ 0 & 5 & -5 & 5 \\ 0 & 0 & 5a+15 & 5b-25 \end{array}\right]$	M1		Use of row operations to create third zero in second column or compare ratios of coefficients in rows 2 and 3
	No unique solution: $5a + 15 = 0$ $a = -3$	A1	3	Solves equation with coefficient of $z = 0$ or equation formed from comparison of ratios (eg $a + 1 = -2$) $a = -3$ is a printed answer
	Alternative 1: $2x - y - z = 3$ ① $x + 2y - 3z = 4$ ② $2x + y + az = b$ ③			
	① + ③ $\Rightarrow$ $4x + (a-1)z = b + 3$ ④ 2③ – ② $\Rightarrow$ $3x + (2a+3)z = 2b - 4$ ⑤	(M1)		Correct elimination of 1 variable. Coefficients must be correct.
	4⑤ –3④ $\Rightarrow$ $(5a + 15)z = 5b - 25$	(M1)		Correctly reduce to one equation with a, b
	$5a + 15 = 0$ $a = -3$	(A1)	(3)	Solves equation with coefficient of $z = 0$
	Alternative 2: Solve $\begin{vmatrix} 2 & -1 & -1 \\ 1 & 2 & -3 \\ 2 & 1 & a \end{vmatrix} = 0$			
	$2\begin{vmatrix} 2 & -3 \\ 1 & a \end{vmatrix} - 1\begin{vmatrix} -1 & -1 \\ 1 & a \end{vmatrix} + 2\begin{vmatrix} -1 & -1 \\ 2 & -3 \end{vmatrix} = 0$	(M1)		Correct expansion by row or column
	$2(2a+3) - (-a+1) + 2(5) = 0$	(M1)		Correct expansion of 2 by 2 determinants
	$5a + 15 = 0$ $a = -3$	(A1)	(3)	Solves equation with determinant = 0

Q	Solution	Marks	Total	Comments
2(b)	Either $5b-25\neq 0$ or $5b-25=0$ Inconsistent $b\neq 5$	M1 A1	2	Sets their constant $\neq 0$ (or 0) CSO (accept $b>5$, $b<5$)
(c)	Linearly dependent since determinant/triple scalar product $=0$ or $\begin{pmatrix}2\\1\\2\end{pmatrix}+\begin{pmatrix}-1\\2\\1\end{pmatrix}+\begin{pmatrix}-1\\-3\\-3\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$	E1	1	Correct deduction with appropriate reason given
	Total		**6**	
3(a)	First factor (quadratic) $= x^2+y^2+z^2$ $\left(x^2+y^2+z^2\right)\begin{vmatrix}x^2-x & y^2-y & z^2-z\\ x & y & z\\ 1 & 1 & 1\end{vmatrix}$	B1		Quadratic factor identified anywhere
	$C_2\to C_2-C_1$ $C_3\to C_3-C_1$ $\left(x^2+y^2+z^2\right)$ $\begin{vmatrix}x^2-x & y^2-x^2-(y-x) & z^2-x^2-(z-x)\\ x & y-x & z-x\\ 1 & 0 & 0\end{vmatrix}$	M1		Correct use of a column/row operation to obtain first linear factor – no more than one slip
	Two linear factors $(y-x)$ and $(z-x)$ $\left(x^2+y^2+z^2\right)(y-x)(z-x)\begin{vmatrix}x^2-x & y+x-1 & z+x-1\\ x & 1 & 1\\ 1 & 0 & 0\end{vmatrix}$	A1,A1		Two correct linear factors found $(y-x)$ $(z-x)$ or equivalent
	Expand to get $\left(x^2+y^2+z^2\right)(y-x)(z-x)(y-z)$	M1 A1	6	Finding the final linear factor $(y-z)$ or equivalent All correct - CSO
(b)	$x,\ y,\ z$ distinct $\Rightarrow\ x\neq y\neq z$ $\Rightarrow\ (y-x)(z-x)(y-z)\neq 0$	E1		Explaining why linear factors are $\neq 0$
	$x, y,\ z$ distinct, real $\Rightarrow\ x^2+y^2+z^2\neq 0$ $\Rightarrow\ \Delta\neq 0$	E1	2	Explaining why the quadratic factor is $\neq 0$
	Total		**8**	

Q	Solution	Marks	Total	Comments
4(a)	direction vector $=\begin{vmatrix} \mathbf{i} & 2 & 1 \\ \mathbf{j} & -2 & 3 \\ \mathbf{k} & 1 & 4 \end{vmatrix} = \begin{pmatrix} -11 \\ -7 \\ 8 \end{pmatrix}$	M1A1		Finding direction of line M1 for attempt at vector product – one component correct
	common point, let $z = 0$ $\left.\begin{array}{l} x-y=12 \\ x+3y=8 \end{array}\right\}$ $\begin{array}{l} y=-1 \\ x=11 \end{array}$	M1A1		Finding common point - M1 for letting $z = 0$ and attempt to solve or equivalent ($x = 0$ gives $y = -8$, $z = 8$ and $y = 0$ gives $x = \frac{88}{7}$ and $z = -\frac{8}{7}$)
	So line is $\dfrac{x-11}{-11} = \dfrac{y+1}{-7} = \dfrac{z}{8}$	A1	5	CAO – any correct equivalent form
	Alternative 1 :			
	Let $z = \lambda$	(M1)		Let $z = \lambda$ and attempt to solve for x, y
	Then $y = -1 - \dfrac{7\lambda}{8}$	(A1)		For y correct
	and $x = 11 - \dfrac{11\lambda}{8}$	(A1)		For x correct
	Gives point $(11, -1, 0)$ and direction $\begin{pmatrix} -11 \\ -7 \\ 8 \end{pmatrix}$	(M1)		Elimination of parameter
	$\Rightarrow \dfrac{x-11}{-11} = \dfrac{y+1}{-7} = \dfrac{z}{8}$	(A1)	(5)	CAO – any correct equivalent form
	Alternative 2 :			
	Let $z = \lambda$	(M1)		Let $z = \lambda$ and attempt to express λ in terms of x, y
	Then $\lambda = \dfrac{8y+8}{-7}$	(A1)		Correct expression in terms of y only
	and $\lambda = \dfrac{8x-88}{-11}$	(A1)		Correct expression in terms of x only
	Hence $\dfrac{8x-88}{-11} = \dfrac{8y+8}{-7} = z$	(M1) (A1)	(5)	Elimination of parameter CAO – any equivalent form
(b)(i)	$\sqrt{11^2 + 7^2 + 8^2} = \sqrt{234}$	M1		Modulus of their direction vector seen and correct structure used for direction cosines
	$\cos\alpha = \dfrac{-11}{\sqrt{234}}$, $\cos\beta = \dfrac{-7}{\sqrt{234}}$, $\cos\gamma = \dfrac{8}{\sqrt{234}}$	A1F	2	ft error in their direction vector

Q	Solution	Marks	Total	Comments
4(b)(ii)	$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$	B1		Seen / stated
	$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$	M1		Trig identity $\cos^2\theta = 1 - \sin^2\theta$ used
	$\Rightarrow 3 - \sin^2\alpha - \sin^2\beta - \sin^2\gamma = 1$			
	$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2$	A1	3	All correct
	Alternative:			
	$\sin^2\alpha = \frac{113}{234}$, $\sin^2\beta = \frac{185}{234}$, $\sin^2\gamma = \frac{170}{234}$	(M1) (A1F)		Attempt to get all of $\sin^2\alpha, \sin^2\beta, \sin^2\gamma$ All correct – ft their direction vector
	$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{113+185+170}{234}$			
	$= \frac{468}{234} = 2$	(B1)	(3)	Correct verification (CSO) – must see explicit calculation to arrive at 2
	Total		**10**	
5(a)	$\begin{vmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 2 \\ -1 & 1 & 3-\lambda \end{vmatrix} = 0$			
	$(1-\lambda)\begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} - \begin{vmatrix} 1 & 2 \\ 2-\lambda & 2 \end{vmatrix} = 0$	M1		Correct row/column expansion of $\lvert \mathbf{M} - \lambda\mathbf{I} \rvert = 0$
	$(1-\lambda)(\lambda-4)(\lambda-1) - 2(\lambda-1) = 0$ $(1-\lambda)\left[(2-\lambda)(3-\lambda) - 2\right] - \left[2 - 2(2-\lambda)\right] = 0$	m1		Correct expansion of 2 by 2 determinants – dependent on first M1
	$\left[\lambda - 1\right]\left[-\lambda^2 + 5\lambda - 6\right] = 0$	A1		or $-\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0$
	$-(\lambda-1)(\lambda-2)(\lambda-3) = 0$	M1		Attempt to show $\lambda = 2$ is an eigenvalue
	$\lambda = 1,\ 2$ or 3	A1	5	Fully correct eigenvalues - CAO
(b)	$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 0 & 2 \\ -1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	M1		Attempt to solve $(\mathbf{M} - 2\mathbf{I})\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$
	$\left.\begin{matrix} \Rightarrow 2z = 0 \\ -x + y + z = 0 \end{matrix}\right] \Rightarrow \begin{matrix} z = 0 \\ x = y \end{matrix}$	A1		Both relationships obtained (can be unsimplified)
	$\Rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector	A1	3	Stated clearly; accept multiples

Q	Solution	Marks	Total	Comments
5(c)(i)	$\begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 0 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	M1		Attempt at $\mathbf{N}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$
	$\Rightarrow \lambda = 4$ is an eigenvalue	A1	2	
(ii)	Eigenvector = $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	B1		Accept multiples – part b) must be correct
	Eigenvalue $= 2 \times 4 = 8$	M1A1	3	M1 for product of relevant eigenvalues
	Alternative for (c)(ii)			
	Eigenvector = $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	(B1)		Accept multiples – part b) must be correct
	$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 2 & -3 \\ 2 & 2 & 3 \\ -3 & 3 & 3 \end{pmatrix} = \begin{pmatrix} -2 & 10 & 6 \\ -2 & 10 & 12 \\ -9 & 9 & 15 \end{pmatrix}$			
	$\begin{pmatrix} -2 & 10 & 6 \\ -2 & 10 & 12 \\ -9 & 9 & 15 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} = 8\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$	(M1)		Multiplies to get MN and attempts to find eigenvalue
	So eigenvalue is 8	(A1)	(3)	Correct eigenvalue
	Total		**13**	

Q	Solution	Marks	Total	Comments
6(a)	Determinant of matrix $= -8+9=1$	M1		Finding determinant and multiplying by area
	Area $= 3\times 1 = 3$ (square units)	A1	2	CAO – must show multiplication or refer to scale factor/invariant area or equivalent
(b)(i)	$\begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ mx+c \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $\Rightarrow$ $(x') = 4x + 3(mx+c)$ $(y') = -3x - 2(mx+c)$	M1		x', y' in terms of x, y, m, c
	Invariant lines $\Rightarrow y' = mx' + c$ $\Rightarrow -3x - 2mx - 2c = 4mx + 3m^2x + 3mc + c$	A1		Use of $y' = mx' + c$
	$\Rightarrow\ 0 = \left(3m^2 + 6m + 3\right)x + 3mc + 3c$ $\Rightarrow\ 3m^2 + 6m + 3 = 0 \quad 3mc + 3c = 0$	M1		Attempt at solving equations where coefficients = 0 or compares coefficients
	$3(m+1)^2 = 0 \qquad 3c(m+1) = 0$ $\Rightarrow\ m = -1 \qquad c$ can be any value	A1		Finding the correct value of m
	$\Rightarrow$ lines are $y = -x + c$	A1	5	Fully correct line – no restriction on c
(ii)	When $c = 0$, $y = -x$ is a line of invariant points	B1	1	Any equivalent form
	SPECIAL CASES – (b)(i)			
	$\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ -x+c \end{pmatrix} = \begin{pmatrix} x+3c \\ -x-2c \end{pmatrix}$ $x' = x + 3c$ $y' = -x - 2c$			SC1 – Correct multiplication as shown
	Consider $-x' + c$ $= -(x+3c) + c$ $= -x - 3c + c$ $= -x - 2c$ $= y'$ Hence $y = -x + c$ is an invariant line			SC2 – correct multiplication as shown above and full algebraic solution using $y' = -x' + c$
	Total		**8**	

Q	Solution	Marks	Total	Comments
7(a)	$\text{Det}(AB) = (4)(1) - (1)(1) = 3 \neq 0$	B1	1	Must state non-zero or $\neq 0$
(b)	Matrix of cofactors $\begin{bmatrix} 0 & 0 & 3 \\ 4 & -1 & 8-4k \\ -1 & 1 & k-8 \end{bmatrix}$	M1 A1		Attempt at matrix of cofactors – at least five correct entries Fully correct matrix of cofactors
	$(AB)^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 4 & -1 \\ 0 & -1 & 1 \\ 3 & 8-4k & k-8 \end{bmatrix}$	M1 A2,1	5	Their cofactor matrix transposed correctly At least five correct = A1 (exclude effect of determinant) All entries fully correct = A2
(c)	$(AB)^{-1} = B^{-1}A^{-1}$ $\Rightarrow B^{-1} = (AB)^{-1}A$			
	$B^{-1} = \frac{1}{3}\begin{bmatrix} 0 & 4 & -1 \\ 0 & -1 & 1 \\ 3 & 8-4k & k-8 \end{bmatrix}\begin{bmatrix} k & 6 & 8 \\ 0 & 1 & 2 \\ -3 & 4 & 8 \end{bmatrix}$	M1 A1		Use of $(AB)^{-1}$ and A multiplied Correct order of multiplication
	$= \frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ -3 & 3 & 6 \\ 24 & -6 & -24 \end{bmatrix}$			
	$= \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 2 \\ 8 & -2 & -8 \end{bmatrix}$	A2,1	4	All correct = A2 5+ correct = A1 (exclude effect of determinant) NB – if an attempt is made to find B by setting up a system of simultaneous equations then M1 – 9 correct equations used A1 for $B = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 2 & \frac{1}{2} \\ \frac{3}{2} & -\frac{1}{2} & -\frac{1}{4} \end{pmatrix}$ Final A2 as above
	Total		**10**	

Q	Solution	Marks	Total	Comments
8(a)	direction ratios of line $= p : 3 : -1$ normal to plane $= 1 : 1 : 2$ $\Big\}$ not equal	B1	1	Accept not parallel or showing vector product is non zero
(b)	$x = 3 + pt$ $y = q + 3t$ $z = 1 - t$	M1		Parametric form seen
	Meets plane $\Rightarrow 3 + pt + q + 3t + 2(1-t) = 10$	A1		Correct substitution in plane
	$\Rightarrow (5+q) + t(p+1) = 10$ Within plane $\Rightarrow q = 5,\ p = -1$	M1A1	4	M1 - Finding one correct value A1 - Both values correct
	Alternative Point (3, q, 1) is common to line and plane			
	Hence $\begin{pmatrix}3\\q\\1\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix} = 10$ which gives $q = 5$	(M1A1)		Uses common point to find q
	Another point common to both is (3 + p, 8, 0) Hence $\begin{pmatrix}3+p\\8\\0\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix} = 10$ which gives p = -1	(M1A1)	(4)	Use of second point and value of q to find p or consideration of scalar product $\begin{pmatrix}p\\3\\-1\end{pmatrix}.\begin{pmatrix}1\\1\\2\end{pmatrix} = 0$
(c)(i)	**n** = $\begin{pmatrix}1\\1\\2\end{pmatrix}$ **d** = $\begin{pmatrix}p\\3\\-1\end{pmatrix}$ Let α be angle between normal and direction ratios (plane) (line)			
	$\mathbf{n}.\mathbf{d} = p + 1$	M1		**n . d** correct
	$\sin\theta = \dfrac{1}{\sqrt{6}} \Rightarrow \cos\alpha = \dfrac{\pm 1}{\sqrt{6}}$	B1		Correct cos α stated or implied
	$\Rightarrow \dfrac{p+1}{\sqrt{6}\sqrt{p^2+10}} = \dfrac{\pm 1}{\sqrt{6}}$	m1 A1		Forming equation connecting all relevant parts **and** attempting to solve for p (condone missing $\pm$) Dependent on first M1 – fully correct for A1
	$\Rightarrow (p+1)^2 = p^2 + 10$ $\Rightarrow p^2 + 2p + 1 = p^2 + 10$ $\Rightarrow 2p = 9$ giving $p = 4.5$	A1	5	CAO

Q	Solution	Marks	Total	Comments
8(c)(ii)	$z = 2 \Rightarrow t = -1 \Rightarrow x = -1.5$ $p = 4.5 \quad y = q - 3$ $\Rightarrow -1.5 + q - 3 + 4 = 10$ $q = 10.5$	M1 A1	 2	Attempt to form an equation for q using t = -1 CAO
	Alternative for (c)(i) $\lvert \mathbf{n} \times \mathbf{d} \rvert = \sqrt{49 + (1+2p)^2 + (3-p)^2}$ $\sin\theta = \frac{1}{\sqrt{6}} \Rightarrow \sin\alpha = \frac{\sqrt{5}}{\sqrt{6}}$ $\frac{\sqrt{49+(1+2p)^2+(3-p)^2}}{\sqrt{6}\sqrt{p^2+10}} = \frac{\sqrt{5}}{\sqrt{6}}$ Leading to $p = 4.5$	(M1) (B1) (m1A1) (A1)	 (5)	$\lvert \mathbf{n} \times \mathbf{d} \rvert$ correct Correct sin α stated or implied Forming equation connecting all relevant parts **and** attempting to solve for p. Dependent on first M1 – fully correct for A1 CAO
	Total		**12**	
	TOTAL		**75**	